Math Fundamentals Problem Packet for Teachers:

Marble Mayhem

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Welcome!

This packet contains a copy of the problem, the "answer check," our solutions, teaching suggestions, and a problem-specific scoring rubric. ***Marble Mayhem*** has been chosen from the Problem Library (#3223), so I'm also including some sample student solutions.

We invite you to visit the PoW discussion groups to explore these topics with colleagues. From the Teacher Office use the link to "PoW Members" or use this URL to go to *funpow-teachers* directly: http://mathforum.org/kb/forum.jspa?forumID=526 [Log in using your PoW username/password.]

The Problem

Marble Mayhem requires understanding the concepts of division as equal sharing, of multiples as repeated addition of the same number, and of common multiples. Young children without experience with the formal vocabulary and procedures involved can successfully solve the problem by using a systematic guess-and-test strategy. Each of the Extra questions is intended to help students generalize the problem and think about it in a deeper way. See the Expected Solutions below.

A print-friendly version is available from the "Print this Problem" link on the problem page.

Marble Mayhem

Fred, Ginger, Julio and Dawn decided to play marbles. Fred emptied his bag of marbles and divided them equally among the four players. Everyone got at least one marble. There was one marble left over.

At that moment Jake arrived and asked to play. They gathered up all Fred's marbles and divided them equally among the five kids. There was still one marble left over.

Just then Maria joined them, so they gathered all the marbles again and divided them equally six ways. There was still one marble left over.

What is the fewest number of marbles that Fred could have had in his bag?

Extra: What is the fewest number of marbles Fred could have had in his bag if Dawn had not been there at all? How did your answer compare with your original answer? Why do you think that is so?

Answer Check

The smallest number of marbles that Fred could have had in his bag is 61.

If your answer doesn't match ours,
- did you understand that each time the marbles were divided, each child got an equal share?
- did you remember that there was always one marble left over after dividing them equally?

If your answer is greater than 61, did you remember to look for the smallest number that would work?

If you used guess-and-test, did you tell . . .
- what numbers you tried?
- how you tested them?
- how you knew whether they worked or not?
- how you decided what to try next?
- about any patterns that helped you?

If your answer does match ours,
- have you clearly shown and explained the work you did?
- did you make any mistakes along the way? If so, how did you find and fix them?
- are there any hints that you would give another student?
- does this problem remind you of experiences you've had?
- did you try the Extra?

Our Solutions

Here are three examples of some ways I imagine children might solve the problem. They are not meant to be prescriptive or comprehensive. In fact, we often receive solutions from students who have used approaches we've not anticipated. Those are cause for celebration! I hope you will share such approaches on the *funpow-teachers* discussion board, along with any teaching strategies you found to be successful.

Strategy 1 – Using manipulatives:

I used inch tiles to represent marbles. When you count by fives, every number ends in zero or five, so the answer has to end in one or six, in order to have one left over. Since I needed the smallest number that works, I started with six tiles and tried to put them in four equal groups, with one left over. It didn't work.

Then I figured out that any number that makes four equal groups has to be an even number. To have one left over, the answer has to have a one in the one's place. I tested 11, but it did not work because you can't make four equal groups out of ten. I could divide 21 into four groups and five groups with one left over, but not six equal groups.

I continued testing numbers that end in one.

- 31 didn't work for four groups.
- 41 worked for four and five, but not six groups.
- 51 didn't work for four groups.
- 61 worked! I could divide it into four, five and six equal groups with one left over each time. It is the smallest number of marbles Fred could have had in his bag. Four children each would have gotten 15 marbles (4 * 15 = 60). Five each would have had 12 marbles (5 * 12 = 60). Six each would have had 10 marbles (6 * 10 = 60).

Extra: If Dawn had not been there, Fred still had to have 61 marbles in his bag for the story to work. I had to find the smallest number that I could divide into three, four and five equal groups with one left over. It still had to end in one because of the four and five equal groups. I thought the answer would be smaller, but I when I used the tiles to test, I found out that 61 was the smallest number that would work. With three children each gets 20 marbles (3 * 20 = 60).

Strategy 2 – Skip Counting:

I knew that the answer had to be one more than a number that had factors of four, five and six. I made a list of numbers you land on when you skip count by fours, fives and sixes. Here they are:

4s	5s	6s
4	5	6
8	10	12
12	15	18
16	20	24
20	25	30
24	30	36
28	35	42
32	40	48
36	45	60
40	50	66
44	55	72
48	60	
52	65	
56	70	
60	75	
64	80	
68		
72		

I began looking down the list of 6s, because there are fewer of them to check. I looked for a number that appeared in all three lists, because that number would have all three factors. Many numbers appeared in two lists but not in all three until I reached 60! It is the smallest number that has factors of 4, 5, and 6. The answer has to be one more that that, or 61.

Extra: If Dawn were not there I would have to find the number that appeared in the lists of 3s, 4s, and 5s. I already had skip counted by 4s and 5s, so I added the list that counts by 3s:

3s: 3, 6, 9, 12, … 54, 57, 60

I was surprised that 60 was the smallest number that appeared in the list of 3s, 4s, and 5s. Fred would have needed 61 marbles in his bag in this case also.

Strategy 3 - Least Common Multiple:

Since the answer, less the one marble left over each time, can be equally divided into 4, 5 and 6 groups, the answer has to be one greater than the least common multiple of 4, 5, and 6. I did a prime factorization of all three numbers:
4's prime factors are 2 * 2.
5's prime factor is 5.
6's prime factors are 2 * 3.

The smallest number that has all those factors is 60. Its prime factors are 2 * 2 * 3 * 5. It contains all the prime factors of 4, 5, and 6. Every time the kids divide the marbles, there is one left over, so the bag must have started with 61 marbles.

Extra: If Dawn was not there, the problem becomes finding the least common multiple of 3, 4 and 5. That is still 60. Fred would still need to have 61 marbles in his bag at the start.

Teaching Suggestions

Children who are developing multiplication concepts will be able to apply them to **Marble Mayhem.** As suggested in Strategy 1, manipulatives can be used to model the problem. Children with more developed concepts of multiples might mark the multiples of 4, 5, and 6 on a 100 grid, using a different color crayon or chip for each multiple. Asking students questions that elicit their understanding about multiples of 5, odd/even numbers, and units digits can help them develop insights that lead to a more efficient approach.

Some students may list numbers that are multiples of 4, 5, and 6 and then add 1 onto the least common multiple, demonstrating good number sense in terms of the affect of the left over marble. Others may list the numbers that are one more than the multiples of 4, 5, and 6, (e.g., 5, 9, 13, 17 for the 4s) and then look for the common number in these lists. While this is a sound strategy, the numbers in these lists are less "friendly" and make it more difficult for students to make use of what they know about the patterns in multiples and final digits. Careful questioning may help move these students toward a more efficient approach.

For students who are ready, **Marble Mayhem** offers an opportunity to develop the understanding of common multiples and least common multiples and to practice using correct math language.

The Teacher Support Page for this problem contains links to related problems in the Problem Library and to other web-based resources: http://mathforum.org/funpow/puzzles/supportpage.ehtml?puzzle=364.

Scoring Rubric

On the last page is the **problem-specific rubric,** to help those who are assessing student solutions. It specifies what we expect from students in three areas of problem solving and three areas of communication. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work. A **generic student-friendly rubric** can be downloaded from the *Scoring Guide* link on every problem page. We encourage you to share it with your students to help them understand our criteria for good problem solving and communication.

Sample Student Solutions

Focus on Interpretation

I've chosen the samples below to illustrate various interpretations demonstrated by submitters. This category of our rubric addresses the student's understanding of the key mathematical ideas in the problem. It is sometimes difficult to assess a student's understanding, especially when little has been written. We try to separate the understanding from the execution of a method. It's unlikely that a student will apply a sound strategy if the understanding is faulty, but, as some of the examples show, it's entirely possible to understand the problem and choose a flawed strategy.

It is not my purpose to give a definitive judgment, but rather to highlight the range and variety of work done by these students and suggest ways I might encourage them to take next steps. See the problem-specific rubric for more details.

Lindsay
Age 11
Interpretation
Novice

7 would be the least amount of marbles Fred would have had.6 would be the least number of marbles if Dawn would not have shown up.

On a piece of paper I drew a different face for Fred,Ginger,Julio and Dawn.First i drew a circle under each face to represent a marble. As I read on Jake came so i drew a face for him and drew a marble under his face. When Maria came i also drew a face for her and drew a marble under her. Then i added up the number of marbles under there faces and got 6,then i added the let over marble nad got 7.

Lindsay has taken pains to write a complete explanation of her strategy. She has missed how the same number of marbles gets redistributed at each stage. I'd ask her to read it again, paraphrase the story, and act it out with manipulatives.

Tim
age 11
Interpretation
Apprentice

There is a total of 61 marbles in Fred's bag.

First i think of how many people there are alotgether. There is six. Now i will guess and check to find my answer. If the four kids got 3 marbles each and there was one left over that would be thirteen marbles. Then Jake comes and divide the marbles with 5 people each person got two marbles theres three left over that does not work. NOw i will try that the first four kids each get 15 marbles. Then Jake comes and each kid get 12 marbles. Finally Maria comes and each kid got ten marbles. That would mean there is 61 marbles total.

Tim understands the story and division as equal sharing. I'm glad he told how many marbles each child gets with a total of 61. Neither his starting number of 13 or his answer of 61 tell us that he understands the "least" requirement. I'd ask him to show that 61 meets it.

Rodney
age 13
Interpretation
Apprentice

The fewest number of marbles that Fred could have had in his bag is 121 marbles.

step 1. since there were first four people, then five, then six, the answer would have to be a multiple of 4, 5, anf 6.

step 2. i multiplied 4x5x6=120

step 3. then i added one since each time it was divided it there was one left

Rodney understands the role of a common multiple and knows how to find one. While he uses "fewest" in his answer, his solution doesn't show that he has considered that. I'd ask him how he could find out whether his answer meets that criterion.

Nick
age 10
Interpretation
Practitioner

The leaast amount of marbles that the children can have is 61.

I decided that I needed to know which number 4,5,and 6 could divide into and have no remainder. I then did least common multiples for the 3 numbers.
4 8 12 16 20 24 28 32 36 40 44 48 52 56 60
5 10 15 20 25 30 35 40 45 50 55 60
6 12 18 24 30 36 42 48 54 60

I then took the next number becase each would have a remainder of 1 if divided into 60.

Nick shows he understands the problem and how finding the LCM will help. By listing all the factors of 4, 5, and 6 in order, he is able to find the lowest common multiple. I'd encourage Nick to try the Extra. He'd be able to take advantage of the work he's done so far, and maybe discover why the LCM is the same.

Teddy
age 11

Interpretation
Practitioner

The fewest amount of marbles Fred could have had in his bag is 61.

First I broke it up into three diagrams. Then I kept on adding marbles to the diagram for when only 4 people were playing until it fit the one where 5 were playing. I did the same with the diagram where 6 people were playing until the same amount of marbles fit each diagram evenly. My answer was that the fewest amount of marbles Fred could have had was 61.

I'd like to see Teddy's diagrams. He shows he understands equal sharing. While he doesn't mention it explicitly, his method does address the "least" criterion. I'd ask him to show calculations to verify his answer and tell how many marbles each child got at each stage.

Woodburn TAG Class
age 9

Interpretation
Expert

For #1 The least number of marbles Fred could have is sixty-one.

#2 If Dawn would not have been there, Fred would have had sixty-one marbles because sixty-one is the only number that is the same between the multiples of 3,4,5.

First, we found the multiples of four , five and six because there were that many players.Then we added one because of the one marble that was left over.Thaen we eliminated the even numbers in the multiples of five plus one becase the multples of four and six plus one were all odd numbers.Finnaly, welooked for a number that was common between all the numbers that were multiples.

The multiples of 4 (plus one) were
5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61, 65, 69, 73, 77 81, 85, 89

The multiples of 5 (plus one) with the multiples ending in 5 eliminated were
11, 21, 31, 41, 51, 61, 71, 81, 91, 101

The multiples of 6 (plus one) were 7, 13, 19, 25, 31, 37, 43, 49, 55, 61, 67, 73, 79, 85, 91

The lowest number that was common to all of these was 81.

When we found the multiples of 3 (plus 1) they were 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43, 46, 49, 52, 55, 58, 61, 64, 67, 70

Again the lowest common number was 61.

This class understands all the key ideas of the problem. Their strategy is sound and results in the fewest marbles. The numbers resulting from adding one onto each multiple are not as "friendly" to think about and find patterns. I'd stretch them to think about whether it was necessary to add the one to each multiple.

Kyle
age 11

Interpretation
Expert

The fewest number of marbles that Fred could have had in his bag is 61.

First there were 4 players, and then became 5 players, and 6 players. So I want to find out the smallest number that is a multiple of 4, 5, and 6. And then you can add left-over 1 to get an answer.

I have to find out the LCM (least Common Multiple) of 4, 5 and 6. Multiply each factor the greatest number of times it occurs in any of the numbers. 4 has two 2s, and 5 has one 5, 6 has 2 and 3.   2X2X3X5=60 ...LSM
I check my work by verifying that 60 can be divided evenly by 4, 5, and 6.

If you add left-over 1 to 60, 61 is my answer.
I check my work by veryfiying that 61 can be divided evenly by 4, 5, and 6, with one left over.
61 / 4 = 15 R1
61 / 5 = 12 R1
61 / 6 = 10 R1

Extra: The fewest number of marbles Fred could have had in his bag if Dawn had not been there is also 61.

I have to find out the LCM of 3,4,and 5.   2X2X3X5=60   And add left-over 1, 61 is my answer.

Both answers are same, because LCMs for both sets of numbers are same.

Kyle understands all the conditions of the problem. He demonstrates a sophisticated method of finding an LCM, and explains it well. I like the fact that he verified his answer, showing how many marbles each child got. He shows why the LCM for both scenarios is the same.

We hope these packets are useful in helping you make the most of FunPoWs. Please let me know if you have ideas for making them more useful.

~ Claire

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The Math Fundamentals Problem of the Week Scoring Rubric — **Marble Mayhem** (posted 17 March 2008)

For each category, choose the level that *best describes* the student's work.

	Novice	Apprentice	Practitioner	Expert
Problem Solving				
Interpretation	Does not show much understanding of the problem.	Shows some understanding of the math in the problem. Completes part of the problem.	Understands the division concept of equal sharing. Understands the multiple concept (repeated addition of same number). Understands one marble left over. Understands "fewest" requirement. Answers the main question.	Solves the main problem correctly. Understands and solves the Extra. Achieves at least Practitioner in Strategy.
Strategy *(NB: based on their interpretation of the problem)*	Does not know how to set up the problem. OR Shows no evidence of strategy. OR Strategy didn't work.	Tries a strategy that makes sense, but isn't enough to solve the whole problem, OR doesn't apply it systematically. OR Verifies a correct answer, but fails to explain how they found it.	Picks a sound strategy. Approaches the problem systematically, achieving success through skill and understanding, not luck. Chosen strategy accounts for any answer(s) that changed after checking our answers.	Does **one or more** of these: Uses two different strategies. Uses a good Extra strategy. Uses an unusual or sophisticated strategy, e.g., effective and appropriate use of technology or algebra.
Accuracy *(NB: based on chosen strategy)*	Has made many errors. OR Shows no math.	Some work is accurate. May have one or two errors. OR Shows very little arithmetic.	Work on main problem is accurate and contains no arithmetic or record keeping mistakes.	Not available for this problem.
Communication				
Completeness *(NB: an incorrect solution can be complete)*	Writes very little to explain how the answer was achieved.	Describes the steps but does not include calculations or numbers. OR Shows calculations without rationale or explanation.	Explains most of the steps taken to solve the problem and the rationale for them, with enough detail for another student to understand. Includes key calculations with rationale. Explains/show why the answer must be the "fewest" possible. Explanation accounts for any answer(s) that changed after checking our answers.	Explains strategy for Extra. Does **one or more** of these: Includes useful extensions and further explanation of concepts or patterns. Provides exceptional insight into the problem.
Clarity *(NB: incomplete and incorrect solutions can be explained clearly*	Explanation is very difficult to read and follow.	Explanation isn't totally unclear, but another student wouldn't be able to follow it easily. Spelling errors/typos make it hard to understand.	Attempts to make explanation readable by a peer. Uses level-appropriate math language and notation, including units: marbles, children. Shows effort to use good organization, formatting, spelling, grammar, typing. Errors don't interfere with readability.	Formatting makes ideas exceptionally clear. Answer is very readable and appealing, might include a helpful table. (A table alone doesn't qualify for Expert status.)
Reflection (See list below.**)**	Does nothing reflective.	Includes one reflective thing.	Includes two reflective things.	Includes 3 or more reflective things or does an exceptional job with 2 of them.
	The items to the right are considered reflective, and could be in the solution OR in the comment they leave after viewing our answer:	• **Revises and improves the submission**. • Checks the answer using a different method. • Explains a hint she/he would give someone.	• Reflects on the reasonableness of the answer. • Connects the problem to prior knowledge/experience. • Describes any errors made and how she/he found and corrected them.	• Comments on AND explains the ease or difficulty of the problem. • Explains where she/he is stuck. • Summarizes the process used.

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